

Noncommutativity density distribution inspired in Einasto density profile

Alberto Hernandez-Almada^{1*} and Miguel A. García-Aspeitia^{2,3†}

¹*Instituto de Física, Universidad Nacional Autónoma de México,
Circuito Exterior C.U., A.P. 20-364, México D.F. 04510, México*

²*Consejo Nacional de Ciencia y Tecnología, Av. Insurgentes Sur 1582. Colonia Crédito Constructor,
Del. Benito Juárez C.P. 03940, México D.F. México and*

³*Unidad Académica de Física, Universidad Autónoma de Zacatecas,
Calzada Solidaridad esquina con Paseo a la Bufa S/N C.P. 98060, Zacatecas, México.
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In this paper we analyze the galaxy rotation curves using a density profile that comes from noncommutativity (NC) theory. We will refer to this distribution as NC density. In this case, we use the Einasto's density profile as a reference due that it is a generalized case of NC distribution and is one of the most successful phenomenological profiles to describe the rotation curve of galaxies. Based on these results, we open a discussion if Einasto's profile could be used as an extension of NC density and if could be applied to other studies treated by this theory.

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I. INTRODUCTION

Nowadays, dark matter (DM) is one of the most elusive problems in modern astrophysics and cosmology, dominating diverse phenomena from rotation curves in galaxy dynamics [1] to large scale structure in the Universe [2–4]. As we mentioned, different observations corroborate its existence, being one of the most significative, that which comes from density profiles and its relation with the velocity rotation of galaxies. In this vein, a natural explanation for this problem is adopt DM as a mechanism to generate a density profile and stabilize spiral galaxies, providing with a weak interacting matter distribution component to explain the observed rotation curves. Thus, the corresponding rotation velocities of galaxies can be explained with several empirical density profiles models based for example on N-body simulations like Navarro-Frenk-White (NFW) [5] or by phenomenological models like pseudo isothermal (PISO) [6], Burkert [7], Einasto [8], among others [9]. Always having in mind that these profiles have their advantages and disadvantages¹, remarking that none of them has the last word until now and the conundrum still insurmountable.

Despite to propose several empirical models of the density profile in galaxies, as far as we know, the microscopic nature of DM remains as a mystery. However in literature, many mechanism propose a plethora of ideas in order to explain DM and its possible relation with the density profiles in galaxies, always having in mind that DM problem is broader than galactic rotation curves.

For instance, supersymmetric models are the more accepted candidate by the scientific community to explain DM, due to its advantages in the standard model of particles (SM) or in quantum gravity theories like Strings [11]. However, several interesting alternatives have been emerging, such as: scalar field DM (SFD) like axions [12, 13] or ultralight scalar fields [14]; even extensions to General Relativity (GR) like $f(R)$ theories [15], brane-world models [16] and NC models [17–21].

From now on, we will focus on NC models and in this sense we are going to discuss briefly the main ideas and validity scales in the following order of cases:

- (a) The first case comes from string theory and is based in the assumption of NC space-time coordinates giving a new version of a gauge theory via the Seiberg-Witten map [17]. In this sense, one of the most commons examples of NC comes from quantum mechanics in two dimensions where in the coordinates operators is encoded the new commutation relations as: $[x^i, x^j] = i\bar{\theta}^{ij}$, $[x^i, p_j] = i\delta_j^i$, $[p_i, p_j] = 0$, where $\bar{\theta}^{ij}$ is the antisymmetric, NC, constant parameter and can be solved assuming $\bar{\theta}^{ij} = \bar{\theta}(\epsilon_{ij}, \epsilon_{ij}, \dots)$ [18, 19], having $\bar{\theta}$ units of length square. In this vein, many studies has been bounded the NC parameter, obtaining values of $\bar{\theta}$ approximately equal to the Planck length [22]. From these ideas, born a Gaussian distribution of minimal width $\sqrt{\bar{\theta}}$ instead of a Dirac-delta function, replacing the point-like structure by smeared objects. This election is not trivial and is due to estimate the amplitude between two states with different mean position by using the Feynman path integral [23].
- (b) In the second case, the NC modifies the form of the energy-momentum tensor presented in GR and as a

* aalmada@fisica.unam.mx

† aspeitia@fisica.uaz.edu.mx

¹ see for example in [10], the issues about how the halo profiles are denser and cuspiest than those inferred observationally

consequence, NC replaces the point-like structures by smeared objects as happens in the first case. Indeed, the usefulness of Gaussian distribution that comes from NC at quantum scales can be straightforwardly extended to the analysis of macroscopic systems like velocity rotation of galaxies or others [21]. In fact, this new approach generates a *bigger* value of the NC parameter than the Planck size obtained in the previous approach; giving expected values of the order of kpc. This is because NC it is associated now to the matter distribution and not to the quantum cells.

Early attempts associated with the second case, can be checked in Ref. [20] leaving aside a robust analysis of the dynamic associated with NC and galaxy rotation curves. In this context, we underscore the study developed by Ref. [21], probing NC in stellar dynamics and galactic rotation curves, bounding the free parameter in both astrophysical cases. In the case that concerns us for this paper is in the galaxy rotation curves developed in this previous study; indeed the authors compare the benefits of NC with other well studied candidates and proposes a catalogue of galaxies to study the galactic rotation curves, leaving open several questions that this article is focus on solving.

Therefore, and under this background this paper will focus only onto the second case, remarking that one important feature is that NC density has similitude with Einasto's profile when the free parameter n is equal to 0.5, allowing a comparison between both profiles. Then, inspired by those ideas we test the Gaussian energy density that comes from NC to describe the galaxy rotation curves and we compare with Einasto profile; arising from them the following question: Is it possible to extend NC density, using the same functional form of Einasto, recovering the success at galactic scales and also recovering the usefulness in black holes? As we show through the paper, this leads us to believe that to the response of this question is affirmative, allowing the possibility to propose and extended NC density profile based in Einasto proposition. Other question which remains open is: this extension can be used in traditional quantum examples of NC to prove its efficiency? i.e., when quantum gravity effects must taken in consideration and not only at macroscopic scales (changing the scale of validity of the free parameter); also we can finally ask, if it is possible to argue about the values of the free parameters, using Einasto's approach?

With those questions in mind, we organize this paper as follows: In Sec. II we present the rotation velocity of NC and Einasto, through the density profile associated for each model. In Sec. III we present the results of NC and Einasto constraints through a estimation of the preferred parameters through the Markov Chain Monte Carlo method and a comparison of those models is performed. Finally, in Sec. IV we open a discussion about of a possible extension of NC using Einasto profile, showing its pros and cons, as well as the benefits in the study of other theories based in NC. In what follows, we work in

units in which $c = \hbar = 1$, unless explicitly written.

II. NONCOMMUTATIVE AND EINASTO ROTATION VELOCITY

This section is aimed to present the densities profiles associated to NC and Einasto with the aim of compare both models with observations in rotation velocities of galaxies.

A. Noncommutativity Profile

It is well known that the rotation velocity at Newtonian level is related to the effective potential and is given by

$$V_{\text{total}}^2(r) = r \left| \frac{d\Phi(r)}{dr} \right| = \frac{G\mathcal{M}(r)}{r}, \quad (1)$$

where \mathcal{M} is the total mass which describes the galactic dynamics. As a good approximation we consider \mathcal{M} as the sum of the disk and DM (halo) components. In other words,

$$V_{\text{total}}^2 = V_{\text{disk}}^2 + V_{\text{halo}}^2. \quad (2)$$

The first ingredient comes from observations and the latter is described by the NC density as [18, 19]:

$$\rho(r)_{\text{NC}} = \rho_0 \exp\left(-\frac{r^2}{4\theta}\right), \quad (3)$$

where $\rho_0 \equiv M/(4\pi\theta)^{3/2}$, M represents NC mass and θ is the free parameter related with the smear of the density distribution. Hence, the NC velocity rotation is expressed as

$$V_{\text{NC}}^2(r) = v_{0\text{NC}}^2 \left(\frac{\sqrt{\theta}}{r} \right) \left| \sqrt{\pi} \text{Erf}\left(\frac{r}{2\sqrt{\theta}}\right) - \frac{r}{\sqrt{\theta}} \exp\left(-\frac{r^2}{4\theta}\right) \right|, \quad (4)$$

where $v_{0\text{NC}}^2 \equiv 4\pi G\theta\rho_0$ and $\text{Erf}(x)$ is the error function.

B. Einasto profile

As was mentioned before, the NC model is a particular case of Einasto's model given by [24]:

$$\rho_E(r) = \rho_{-2} \exp\left\{-2n \left[\left(\frac{r}{r_{-2}} \right)^{1/n} - 1 \right] \right\}. \quad (5)$$

The r_{-2} is the radius where the density profile has a slope -2 and ρ_{-2} is the local density at that radius; the parameter n is known as Einasto index which describes the shape of the density profile.

It is also possible to note that from Eq. (1) the rotation velocity can be written as:

$$V^2(r) = v_{0E}^2 \left(\frac{r_s}{r} \right) \gamma \left(3n, 2n \left(\frac{r}{r_{-2}} \right)^{1/n} \right), \quad (6)$$

where $v_{0E}^2 = 4\pi G n r_s^2 \rho_{-2} \exp(2n) (2n)^{-3n}$ and $\gamma(a, x) = \int_0^x t^{a-1} e^{-t} dt$, is the incomplete gamma function. Notice that when $n = 0.5$ we recover the functional form of Eq. (4).

In order to compare both profiles, we may identify that the NC density in the core ρ_0 is related to $\rho_{-2} e^{2n}$ in Einasto case, and the smear parameter $\sqrt{\theta}$ has an equivalent to $r_{-2}/2$.

III. RESULTS

In this section, we describe the procedure to obtain the best fit to rotation curve (RC) data and explore the confidence region. We model the RC distribution as the sum of a stellar disk and a spherical dark halo component. The disk component is modeled by a non-parametric smooth curve and the latter is taking account the NC or Einasto models.

In this vein, we use the Markov Chain Monte Carlo (MCMC) method implemented in the pymc package [25] to estimate the confidence region of the model parameters, $\Theta = (\rho_0, \sqrt{\theta})$ or (ρ_{-2}, r_{-2}) respectively. In addition, the best fit is obtained by maximizing the likelihood function $\mathcal{L}(\Theta) \propto \exp[-\chi^2(\Theta)/2]$, where

$$\chi^2(\Theta) = \sum_i^N \left(\frac{V_{\text{obs}}^i - V_{\text{total}}(\Theta)}{dV_{\text{obs}}^i} \right)^2, \quad (7)$$

here $V_{\text{obs}}^i \pm dV_{\text{obs}}^i$ is the observed velocity and its corresponding uncertainty at the radial distance r_i .

In order to study the same group of galaxies, we focus in galaxies of the type NGC which are enumerated in Tables I and II. Indeed, the reader can observe that we choose those galaxies because they are restricted to the same observational parameters.

For each galaxy, we generate 15k chains using an adaptive Metropolis step method to explore the confidence region. The iteration starts with a random values and is taken off a period of 300 chains (burn-in) to stabilize the steps. In addition, the priors on all parameters are flat distributions.

The Tables I and II are the results of the best fits for each model, NC and Einasto respectively. Also, the reduced χ^2 is presented, which is defined by $\chi_{\text{red}}^2 = \chi^2/(N - p - 1)$, where N is the total number of data and p is the number of free model parameter. In order to compare the results of these models, we present for the Einasto case, the density evaluated at $r = 0$ and $r_{-2}/2$ as an equivalent to $\sqrt{\theta}$ parameter.

In Figs. 1, the best fits to data are drawn with dashed lines corresponding to NC and solid lines for Einasto.

From here it is possible to notice that the latter model produces better fits than NC, that is confirmed by comparing their corresponding χ_{red}^2 .

Also, we choose NGC2366, NGC3521 and NGC7793, a sample of galaxies to present Figs. 2 as the comparison of the parameter phase-space obtained in each model on the plane ρ_0 - $\sqrt{\theta}$ with $n \leq 1.3$ in Einasto model. Notice that in case $n = 1.5$, Einasto model is reduced to NC. The contours represent 1σ and 2σ of confidence levels respectively. In addition, we also present in Figs. 3, the correlations of the free parameters of both theories respectively, showing the confidence region for the samples of galaxies.

As a complement of the study, we test the invariant of NC which is its total mass; the results are plotted in Fig. 4 for the set of galaxies used. The results are computed using the values of ρ_0 and $\sqrt{\theta}$ obtained from Table I.

Galaxy	χ_{red}^2	ρ_0 ($10^2 M_{\odot}/pc^3$)	$\sqrt{\theta}$ (kpc)
NGC2366	0.22	5.54 ± 0.48	1.44 ± 0.08
NGC2403	9.78	5.02 ± 0.09	3.83 ± 0.03
NGC3031	4.83	14.50 ± 0.57	2.31 ± 0.05
NGC3198	8.98	1.77 ± 0.05	6.88 ± 0.09
NGC3521	6.91	4.10 ± 0.25	5.10 ± 0.17
NGC3621	7.95	2.26 ± 0.06	6.06 ± 0.06
NGC6946	4.95	6.67 ± 0.07	3.72 ± 0.07
NGC7331	2.50	2.74 ± 0.19	6.61 ± 0.26
NGC7793	3.44	14.40 ± 0.37	1.75 ± 0.03

TABLE I: For NC model, from left to right: Name of LSB galaxies under study, the reduced χ^2 , NC energy density and the NC parameter.

Galaxy	χ_{red}^2	$\rho_{-2} e^{2n}$ ($10^2 M_{\odot}/pc^3$)	$r_{-2}/2$ (kpc)	n
NGC2366	0.35	$4.66^{+4.40}_{-2.23}$	$1.66^{+0.19}_{-0.16}$	$0.85^{+0.14}_{-0.15}$
NGC2403	1.25	$(1.04 \pm 0.01) \times 10^8$	$4.43^{+0.02}_{-0.04}$	3.17 ± 0.01
NGC3031	3.98	$919.0^{+159.0}_{-280.0}$	1.91 ± 0.07	$1.48^{+0.03}_{-0.05}$
NGC3198	0.39	$(5.98^{+0.16}_{-0.02}) \times 10^7$	$7.08^{+0.36}_{-0.03}$	3.02 ± 0.01
NGC3521	6.81	$43.9^{+35.9}_{-20.4}$	5.37 ± 0.32	$1.30^{+0.09}_{-0.11}$
NGC3621	1.92	$344.0^{+13.3}_{-25.1}$	$6.85^{+0.17}_{-0.15}$	1.69 ± 0.01
NGC6946	2.46	$608.0^{+23.9}_{-60.1}$	4.22 ± 0.15	1.62 ± 0.02
NGC7331	1.30	$897.0^{+149.0}_{-257.0}$	$6.90^{+0.38}_{-0.45}$	$1.77^{+0.31}_{-0.05}$
NGC7793	3.43	$21.9^{+14.9}_{-7.5}$	$1.93^{+0.07}_{-0.06}$	$0.96^{+0.10}_{-0.09}$

TABLE II: For Einasto model, from left to right: Name of LSB galaxies under study, the reduced χ^2 , the density at $r = 0$, $r_0 = r_{-2}/2$, and the spectral index n .

IV. DISCUSSION

We give the task of study NC as a candidate to reproduce the velocity rotation of galaxies for DM halos and compare with one of the most successful candidates (Einasto's model), that is a generalized profile containing

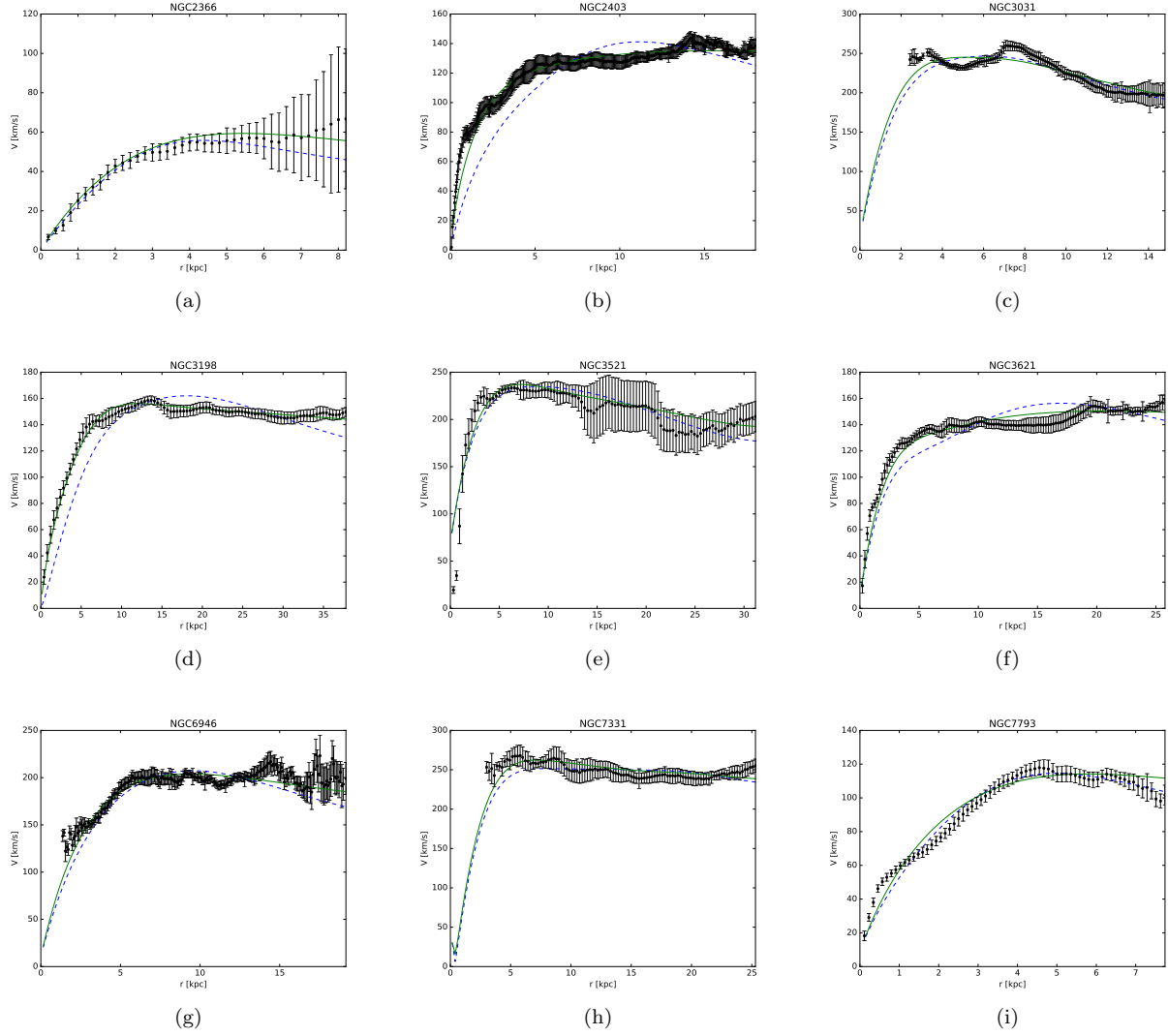


FIG. 1: Set of galaxies analyzed and best fit obtained using the NC (dashed line) or Einasto (solid line) profile. The total model is considering Eq. (2).

the first one. Due to the effort for understanding the DM fundaments, it is notorious that among the plethora of models for describing the galaxy halo, NC is one of the most suggesting candidates because its theoretical fundaments comes from string theory based in the idea of a noncommutative space-time. Besides these advantage, from our study, it is possible to infer that NC does not model in a perfect sense the observations of galaxy rotation velocities in comparison with Einasto profile. Indeed, the discussion of the results can be summarized in the following two paragraphs:

According to the results presented in Tables I and II, we observe the χ^2_{red} values obtained for the galaxies analyzed, Einasto's model has a better agreement to data than NC. However, both models presents similar shape in the most of galaxies such as: NGC2366, NGC3031, NGC3521, NGC7793, NGC6946, and NGC7331. Notice that except of the last two of them, they have similar values of χ^2_{red} .

Also, for spectral index $n \leq 1.3$, the agreement in the parameters ρ_0 and $\sqrt{\theta}$ between models are at 1 sigma. In Fig. 2 are the comparison of the contours at 1 and 2 sigmas. On the other hand, the greater discrepancy in ρ_0 are when $n > 3$, being the cases of NGC2403 and NGC3198 with differences of order of 10^8 and 10^7 between models, respectively. Notice the lower values of ρ_0 reported for NC which present a bias due to the disagreement to the best fit (see Fig. 1) to data.

Despite the unfavorable results from NC model, one question enters to the scene: Is it possible to extend NC density distribution to a functional form like Einasto?

What are the advantages and disadvantages? If this is the case, what the interpretation of the spectral index (n). Then, we suggest a natural extension of noncommutativity written as:

$$\rho_{NC}(r) = \rho' \exp \left\{ -2n \left[\left(\frac{r}{4\sqrt{\theta}} \right)^{1/n} - 1 \right] \right\}, \quad (8)$$

where ρ' is redefined as: $\rho' \equiv M/(4\pi\theta e^{2/3})^{3/2}$. It is important to notice that Eq. (8) must retrieve the success of velocity rotation at galactic scales. However, we remark that are also inherited the same behavior of $r_{-2} \rightarrow \sqrt{\theta}$ and $\rho_{-2} \rightarrow \rho'$ in the core, in contrast, with the expected values for a Gaussian distribution.

Another important goal to explore is the possible usefulness at quantum gravity scales, adjusting now the new free parameter n , always having in mind that Eq. (8) recover the conventional NC structure when $n = 0.5$.

From here we suggest the exploration of NC using this new approach, starting with the collapse and stability of stellar structures, black holes and Hawking thermal radiation, among others. However this is research that will be reported elsewhere.

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- [1] Y. Sofue and V. Rubin, *Annu. Rev. Astron. Astrophys.* **39**, 137 (2001).
 - [2] Planck Collaboration *et al.*, arXiv:1303.5076.
 - [3] C. S. Frenk and S. D. M. White, *Ann. Phys.* **507** (2012), arXiv:1210.0544v1 [astro-ph].
 - [4] A. Diaferio, in *From the vacuum to the universe. Proceedings, Symposium, Innsbruck, Austria, October 19-20, 2007* (2008) pp. 71–85, arXiv:0802.2532 [astro-ph].
 - [5] J. F. Navarro, C. S. Frenk, and S. D. M. White, *Astrophys. J.* **490** (1997).
 - [6] K. G. Begeman, A. H. Broeils, and R. H. Sanders, *Mon. Not. R. astr. Soc.* **294** (1991).
 - [7] A. Burkert, *Astrophys. J. Lett.* **L25** (1995).
 - [8] J. Einasto, *Trudy Inst. Astrofiz. Alma-Ata* **87** (1965).
 - [9] W. Dehnen, *Mon. Not. Roy. Astron. Soc.* **265**, 250 (1993).
 - [10] J. Navarro, C. Frenk, and S. White, *ApJ* **490**, 493 (1997); R. C. K. Subramanian and J. Ostriker, *ibid.* **538**, 528 (2000); G. Gentile, P. Salucci, U. Klein, D. Vergani, and P. Kalberla, *Mon. Not. Roy. Astron. Soc.* **351**, 903 (2004).
 - [11] P. Horava and E. Witten, *Nucl. Phys.* **B475**, 94 (1996), arXiv:hep-th/9603142 [hep-th]; *Nucl. Phys.* **B460**, 506 (1996), arXiv:hep-th/9510209 [hep-th].
 - [12] J. weon Lee and I. gyu Koh, *Phys. Rev.* **D53**, 2236 (1996), arXiv:hep-ph/9507385 [hep-ph].
 - [13] J. Barranco and A. Bernal, *Phys. Rev.* **D83**, 043525 (2011), arXiv:1001.1769 [astro-ph.CO].
 - [14] L. A. Urena-Lopez and T. Matos, *Phys. Rev.* **D62**, 081302 (2000), arXiv:astro-ph/0003364 [astro-ph].
 - [15] C. F. Martins and P. Salucci, *Mon. Not. R. Astron. Soc.* **381**, 1103 (2007).
 - [16] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999), arXiv:hep-ph/9905221; *Phys. Rev. Lett.* **83**, 4690 (1999), arXiv:hep-th/9906064 [hep-th]; R. Maartens and K. Koyama, *Living Rev. Rel.* **13**, 5 (2010), arXiv:1004.3962 [hep-th]; M. A. Garcia-Aspeitia and M. A. Rodriguez-Meza, (2015), arXiv:1509.05960 [gr-qc].
 - [17] N. Seiberg and E. Witten, *JHEP* **09**, 032 (1999), arXiv:hep-th/9908142 [hep-th].
 - [18] A. Smailagic and E. Spallucci, *J. Phys* **A37**, 7169 (2004).
 - [19] P. Nicolini, A. Smailagic, and E. Spallucci, *Phys. Lett.* **B632**, 547 (2006), arXiv:gr-qc/0510112 [gr-qc].
 - [20] F. Rahaman, P. K. F. Kuhfittig, K. Chakraborty, A. A. Usmani, and S. Ray, *Gen. Rel. Grav.* **44**, 905 (2012), arXiv:1011.1538 [gr-qc].
 - [21] M. A. Garcia-Aspeitia, J. C. Lopez-Dominguez, C. Ortiz, S. Hinojosa-Ruiz, and M. A. Rodriguez-Meza, (2015), arXiv:1511.06740 [gr-qc].
 - [22] J. M. Romero and J. D. Vergara, *Mod. Phys. Lett.* **A18**,

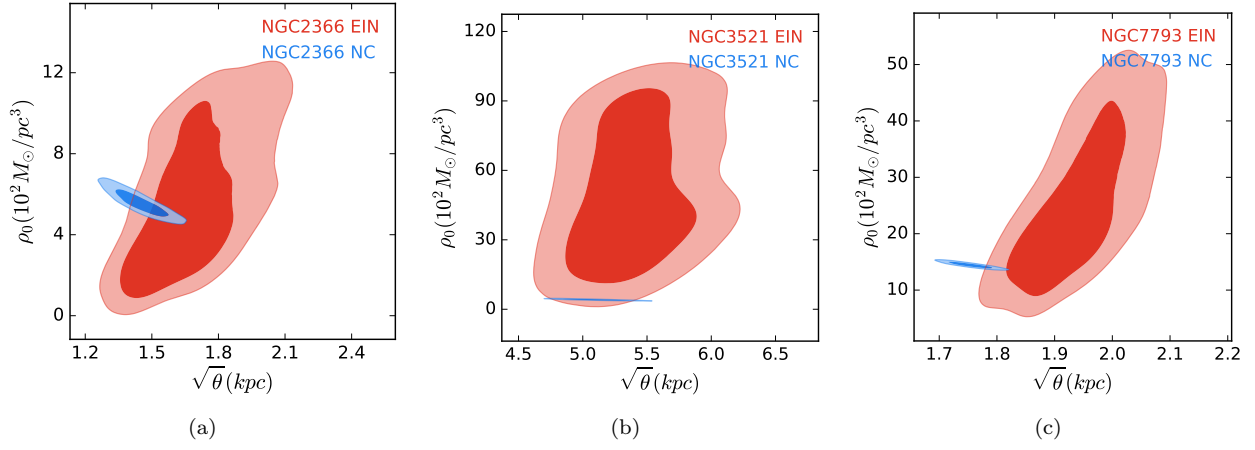


FIG. 2: Samples of galaxies NGC2366, NGC3521 and NGC7793, showing the comparison of contours for NC and Einasto profiles in the parameters: core density ρ_0 and $\sqrt{\theta}$, $r_{-2}/2$. In these galaxies the mean value of the spectral index in the Einasto model is $n \leq 1.3$.

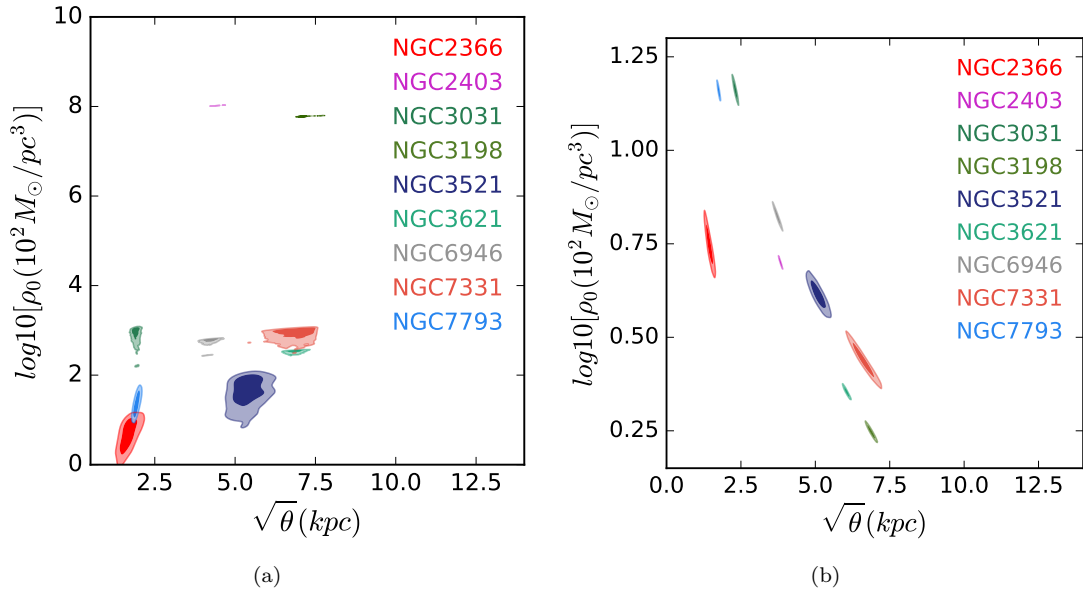


FIG. 3: Correlation of the parameters (ρ_0 and $\sqrt{\theta}$ or $r_{-2}/2$) for: Einasto (left) and NC (right).

- 1673 (2003), arXiv:hep-th/0303064 [hep-th].
- [23] A. Smailagic and E. Spallucci, J. Phys. **A36**, L467 (2003), arXiv:hep-th/0307217 [hep-th]; Journal of Physics A: Mathematical and General **36**, L517 (2003).
- [24] J. Einasto, Trudy Inst. Astrofiz. Alma-Ata **17** (1965).
- [25] A. Patil, D. Huard, and C. J. Fonnesbeck, J. Stat. Softw., 1 (2010).

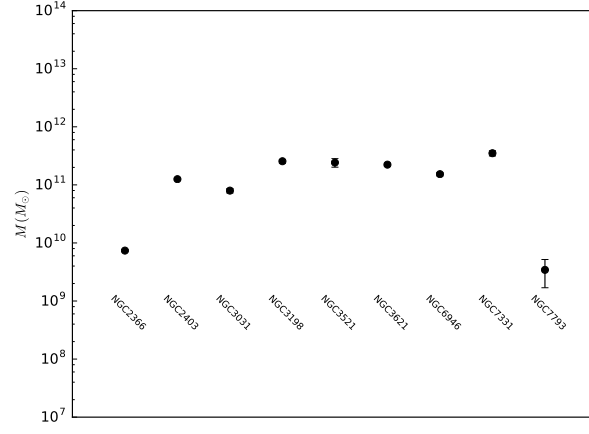


FIG. 4: NC mass plot using $M \equiv \rho_0(4\pi\theta)^{3/2}$; which is associated with the constant parameter of the theory. In this case, we plot the NC mass expected for the group of LSB galaxies studied during the paper. See the text for more details.